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Published in:
IEEE Control Systems Letters

DOI (link to publication from Publisher):
10.1109/LCSYS.2020.3003238

Publication date:
2021

Document Version
Accepted author manuscript, peer reviewed version

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/LCSYS.2020.3003238, IEEE Control Systems Letters

Fault Isolation in MIMO Systems based on Active Decoupling

Henrik Niemann, Jakob Stoustrup, Niels Kjølstad Poulsen

***Abstract*—A decoupling approach for fault isolation in MIMO systems is presented in this paper. The fault isolation approach is based on a closed-loop concept, where the feedback controller is an integrated part of the set-up. The YJBK-parameterization (after Youla, Jabr, Bongiorno, and Kucera) for controllers is introduced. This allows the feedback controller to be modified by changing the YJBK matrix transfer function without changing the nominal feedback controller.**

Modification of the feedback controller via the YJBK transfer matrix causes the output residual response to be changed in the case of parametric faults and unchanged in the fault-free case. This facilitates fault detection. In the isolation case, the controller is modified in a way such that the residual response from the closed-loop system is independent of the modification for one specific parametric fault and else it depends on the parametric faults. This allows for any specific fault and eventually for all faults to be isolated.

***Index Terms*—Fault diagnosis, linear systems, control system architecture.**

I. Introduction

FAULT diagnosis has been an active research area in many years. The reason is the increasing complexity of technical systems used today. Reliability gets more and more important in these systems. We need to be able to guarantee closed-loop stability and in some cases also a certain level of performance. Fault diagnosis is one of the central elements in many approaches to obtain fault-tolerant and reliable systems.

Fault diagnosis can be based on passive methods or active methods. In passive methods, the fault diagnosis is based on naturally available input and output signals in the system, whereas active excitation of the system is also included in active methods. Fault diagnosis methods based on the passive approach has been investigated in a large number of publications, see e.g. the books [1], [2], [4], [5] and the references therein. Fault diagnosis based on the active approach has not been investigated to the same level. One of the first fault diagnosis methods based on the active approach was developed by Zhang, [18]. Later, other approaches have been developed, [3], [8], [10]. Some new results can be found in [6], [9], [17].

Active use of feedback controllers in connection with fault diagnosis has been described in [12], [15]. The presented concept is that the controller is modified to optimize the fault diagnosis task. In [15], the controller is modified such that the closed-loop system is rendered temporarily unstable when faults have occurred in the system and turn stable in the fault free case. For safety reasons, this method has limited applicability. In the approach described in [12], the controller is modified by a change of the YJBK matrix transfer function in the YJBK architecture. By doing this, it is possible to optimize the sensitivity to detecting parametric faults using active methods.

The main contribution in this paper is to develop a fault detection and isolation method using the controller modification principle described in [12]. It is shown that changing the feedback controller via the YJBK matrix transfer function will change the matrix transfer function from external input to the residual output in the faulty case, whereas it will be unchanged in the fault free case. This gives a simple method to detect parametric faults in the system. How much the controller needs to be modified depends on requirements for the time to detect.

The isolation task is more complex than the detection task. In this paper we will use a decoupling approach for fault isolation. The concept from the fault detection is again used for fault isolation. Some YJBK matrix transfer functions are designed. Each YJBK matrix transfer function is designed such that the residual output will be changed for all parametric faults except for a single fault. By switching between the different YJBK matrix transfer functions, it is possible to isolate single parametric faults. A simple rank condition is given for guarantees of fault isolation.

The rest of this paper is organized as follows. In Section II, the system set-up is given together with some results for YJBK parameterization. Section III describe the fault detection problem based on decoupling followed by Section IV, where the isolation problem is considered. An example is given in Section V. The paper is closed with a conclusion in Section VI.

A. Notation

Let X be a matrix of some dimension (n,m). Then, x_{ij} denotes the (i,j) element of X, $X_{i:}$ is the $i'th$ row of X and $X_{:j}$ is the $j'th$ column of X. The nullspace of X is given by $\mathcal{N}(X)$. A lower LFT is defined by $\mathcal{F}_l(P,K) = P_{11} + P_{12}K(I-P_{22}K)^{-1}P_{21}$. An upper LFT is defined by $\mathcal{F}_u(G,\theta) = G_{22} + G_{21}\theta(I-G_{11}\theta)^{-1}G_{12}$.

II. System Set-up

Let the system to be considered be given by a frequency domain model of the form:

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$$\Sigma : \begin{cases} z = G_{zw}w + G_{zd}d + G_{zu}u \\ y = G_{yw}w + G_{yd}d + G_{yu}u \end{cases} \tag{1}$$

$d \in \mathbb{R}^r$ is an external input vector (e.g. reference inputs, disturbance inputs, and measurement noise), $u \in \mathbb{R}^m$ the control input vector and $y \in \mathbb{R}^p$ is the measurement vector. Further, $w \in \mathbb{R}^k$ and $z \in \mathbb{R}^k$ are external input and output vectors. Let the system be controlled by a stabilizing feedback controller given by:

$$\Sigma_C : \{ u = Ky \tag{2}$$

The parametric faults are modeled by θ_i, $i = 1, \cdots, k$, where k is the number of possible parametric faults. Further, let θ represent all the k parametric faults. Let θ be a diagonal matrix, where the (i,i) element of θ given by θ_i is the $i'th$ parametric fault in the system. The fault free case is given by $\theta = 0$. The connection between z and w is given by:

$$w = \theta z \tag{3}$$

Closing the loop from w to z in Σ by using θ, the resulting system can be realized by an upper linear fractional transformation (LFT) in θ given by (see [14], [19]):

$$\Sigma_\theta = \mathcal{F}_u(\Sigma, \theta)$$

where Σ_θ is given by:

$$\Sigma_\theta : \{ y = G_{yd}(\theta)d + G_{yu}(\theta)u \tag{4}$$

A. The YJBK Parameterization

Let a coprime factorization of G_{yu} from (1) and the stabilizing controller K from (2) be given by:

$$\begin{array}{lll} G_{yu} & = NM^{-1} = \tilde{M}^{-1}\tilde{N}, & N, M, \tilde{N}, \tilde{M} \in \mathcal{RH}_\infty \\ K & = UV^{-1} = \tilde{V}^{-1}\tilde{U}, & U, V, \tilde{U}, \tilde{V} \in \mathcal{RH}_\infty \end{array} \tag{5}$$

where the eight matrices in (5) must satisfy the double Bezout equation given by, see [16]:

$$I = \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} \begin{pmatrix} M & U \\ N & V \end{pmatrix} = \begin{pmatrix} M & U \\ N & V \end{pmatrix} \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} \tag{6}$$

Based on the factorization of the system G_{yu} and the controller K in (5), a parameterization of all controllers that stabilize the system in terms of a stable matrix transfer function Q, i.e. all stabilizing controllers are given by a lower LFT in Q:

$$K(Q) = \mathcal{F}_l\left(\begin{pmatrix} UV^{-1} & \tilde{V}^{-1} \\ V^{-1} & -V^{-1}N \end{pmatrix}, Q \right) = \mathcal{F}_l(J_K, Q) \tag{7}$$

The set-up for the YJBK parameterized feedback controller $K(Q)$ is shown in Fig. 1.

In the same way, it is possible to derive a parameterization in terms of a stable matrix transfer function S of all systems that are stabilized by one controller, i.e. the dual YJBK parameterization. A lower LFT representation of the parameterization is given by [16]:

$$\begin{aligned} G_{yu}(S) &= \mathcal{F}_l\left(\begin{pmatrix} NM^{-1} & \tilde{M}^{-1} \\ M^{-1} & -M^{-1}U \end{pmatrix}, S \right) \\ &= \mathcal{F}_l(J_G, S) \end{aligned} \tag{8}$$

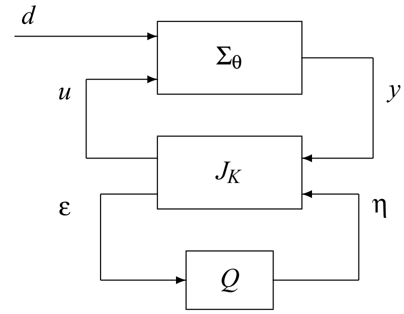

Fig. 1. The setup for the YJBK parameterization including the external disturbance.

Further, S is given by, [16]:

$$S = \mathcal{F}_u(J_K, G_{yu}(S)) \tag{9}$$

The dual YJBK matrix transfer function S is a function of the system deviation from nominal. Here we will only consider parametric variations in terms of the parametric faults described by θ, i.e. $S = S(\theta)$. Assuming that $\theta = 0$, i.e. the nominal value of θ, the following simple relation exist, [10]:

$$S(\theta) = 0, \text{ for } \theta = 0$$

This relation is the central element in the active fault diagnosis approach described in [10], [11]. By testing, if $S(\theta)$ is zero or non-zero, parametric faults can be detected using an active approach.

III. Detection of Parametric Faults

Consider the setup shown in Fig. 1 without Q as shown in Fig. 2. The output ε is a residual vector as shown in [10] for using in connection with fault diagnosis given by:

$$\varepsilon = \tilde{M}y - \tilde{N}u \tag{10}$$

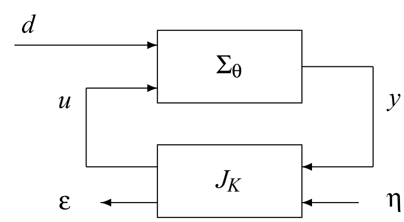

Fig. 2. The feedback controller K including the residual vector ε and the input vector η.

From (9) we have that S is the matrix transfer function from η to the residual vector ε. The open-loop residual vector ε is then given by, [10]:

$$\varepsilon = P_{\varepsilon d}(S)d + S(\theta)\eta = (\tilde{M} + S(\theta)\tilde{U})G_{yd}(\theta)d + S(\theta)\eta \tag{11}$$

The residual vector ε will be named as the open-loop residual vector in the following. A corresponding closed-loop residual vector will be defined below. Further, $S(\theta)$ is also named the *fault signature matrix*, [10] in connection with active fault diagnosis in closed-loop systems.

In the fault free case, ($S(0)=0$), the open-loop residual vector ε in (11) is given by:

$$\varepsilon = \tilde{M}G_{yd}(0)d \tag{12}$$

For fault detection based on a controller change, closing the lower loop around J_K as shown in Fig. 2 by using

$$\eta = Q\varepsilon$$

gives the controller shown in Fig. 3

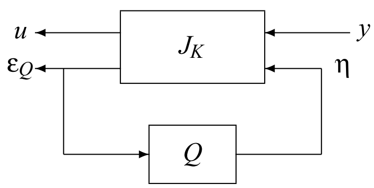

Fig. 3. The YJBK parameterized feedback controller $K(Q)$ including the residual vector ε_Q as an external output from the feedback controller.

The closed-loop residual output vector ε_Q is given by:

$$\varepsilon_Q = (I - S(\theta)Q)^{-1}P_{\varepsilon d}(S)d = S_Q(\theta)P_{\varepsilon d}(S)d \tag{13}$$

The closed-loop residual vector ε_Q depends on the external input d, the parametric fault θ, the nominal system, the feedback controller and the YJBK matrix transfer function Q.

The closed-loop residual vector ε_Q given by (13) can now be analyzed with respect to disturbance input d and parametric faults θ. For the nominal case, ε_Q is given by:

$$\varepsilon_Q = \varepsilon \text{ for } d \neq 0,\ \theta = 0,\ \forall Q \tag{14}$$

In the faulty case, ε_Q is given by:

$$\varepsilon_Q \neq \varepsilon \text{ for } d \neq 0,\ \theta \neq 0,\ \forall Q \in Q_g,\ S(\theta)Q \neq 0 \tag{15}$$

where Q_g has a complement of zero measure.

An important observation from (14) is that the closed-loop residual vector ε_Q is independent of a change in the controller via Q in the fault-free case. Further, from (15), we have that ε_Q depends on Q in the faulty case. This makes it possible to discriminate between the effect from the external input d and parametric faults, i.e. detection of parametric faults in the system. It is also important to point out that $S(\theta)$ does not require to be given explicitly for the controller based detection of parametric faults. Further, note that the detection results can also be applied to SISO systems.

IV. Parametric Fault Isolation

It will require unique signatures in the residual vector with respect to the different parametric faults to be able to isolate them. As in the detection case, the isolation is also based on a non-zero unknown external input d. As a consequence of d being unknown, a change of the feedback controller with a specific Q will not give a well defined change in the residual vector for a specific parametric fault. Instead, the isolation needs to be done indirectly. One method for obtaining a unique signature or a unique signature change in the residual vector is to design a dedicated Q_i for each specific parametric fault θ_i. Let Q_i be designed such that ε_Q is unchanged when θ_i occur in the system and changed when $\theta_j,\ j \neq i$ occur, i.e. the closed-loop residual vector ε_Q will satisfy:

$$\begin{aligned} \varepsilon_Q(\theta_i) &= \varepsilon(\theta_i) \quad \text{for} \quad Q = Q_i \neq 0, \quad \theta_i \neq 0 \\ \varepsilon_Q(\theta_j) &\neq \varepsilon(\theta_j) \quad \text{for} \quad Q = Q_i \neq 0, \quad \theta_j \neq 0, \quad i \neq j \end{aligned} \tag{16}$$

Based on (13), (16) is satisfied if Q_i satisfy the following design condition:

$$\begin{aligned} S(\theta_i)Q_i &= 0, \quad Q_i \neq 0, \quad \theta_i \neq 0 \\ S(\theta_j)Q_i &\neq 0, \quad Q_i \neq 0, \quad \theta_j \neq 0, \quad i \neq j \end{aligned} \tag{17}$$

For satisfying the first condition in (17), Q_i must be selected such that it is in the nullspace of $S(\theta_i)$, i.e. $Q_i \in \mathcal{N}(S(\theta_i))$. The second condition is satisfied if Q_i is not in the nullspace of $S(\theta_j)$. This require that $\mathcal{N}(S(\theta_j)) \neq \mathcal{N}(S(\theta_i)),\ j \neq i$. For a more detailed analysis of the design conditions given by (17), let us consider the fault signature matrix $S(\theta)$ in more details.

Based on the LFT description of $G_{yu}(\theta)$ in (4), the fault signature matrix $S(\theta)$ is then given by, [10]:

$$\begin{aligned} S(\theta) &= \tilde{M}G_{yw}\theta(I - (G_{zw} + G_{zu}U\tilde{M}G_{yw})\theta)^{-1}G_{zu}M \\ &= T_1\theta(I - T_2\theta)^{-1}T_3 \end{aligned} \tag{18}$$

Assume that θ_i is the only non-zero element in θ, then $S(\theta_i)$ takes the following form:

$$S(\theta_i) = \tfrac{\theta_i}{1 - t_{2,ii}\theta_i} T_{1,:i}T_{3,i:} \tag{19}$$

The first design condition in (17) is then given by:

$$\begin{bmatrix} t_{1,1i} \\ \vdots \\ t_{1,ii} \\ \vdots \\ t_{1,pi} \end{bmatrix} \begin{bmatrix} t_{3,i1} & \cdots & t_{3,ii} & \cdots & t_{3,im} \end{bmatrix} Q_i = 0 \tag{20}$$

where the transfer function $\theta_i/(1 - t_{2,ii}\theta_i)$ has been removed from $S(\theta_i)$ because it is a scalar non-zero transfer function.

The condition given by (20) is now independent of θ_i. Further, when it is assumed that the fault θ_i is detectable, we will have that $S(\theta_i)$ is non-zero and therefore $T_{1,:i}T_{3,i:}$ will have *normal rank* 1. The *normal rank* of a system $G(s)$ is the maximal rank of $G(s)$, apart from a finite number of values for s where the rank is reduced, [7]. Q_i is of dimension (m, p). It is clear that if (and only if) m is larger than 1, i.e. there is more than one control signal in the system, there will exist a non-zero Q_i satisfying the above equation.

The condition can be reduced further, because $T_{1,:i}$ has full column rank and can therefore be removed from (20). This gives the following simple condition for the design of Q_i:

$$T_{3,i:}Q_i = \begin{bmatrix} t_{3,i1} & \cdots & t_{3,ii} & \cdots & t_{3,im} \end{bmatrix} Q_i = 0 \tag{21}$$

Q_i just need to be designed such that it is in the nullspace of $T_{3,i:}$ given by:

$$Q_i \in \mathcal{N}(T_{3,i:}) = \mathcal{N}((G_{zu}M)_{i:}) \tag{22}$$

The second condition in (17) gives that Q_i also need to satisfy:

$$Q_i \notin \mathcal{N}(T_{3,j:}),\ j \neq i \tag{23}$$

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(22) together with (23) gives the following simple condition for Q_i to satisfy for an isolation of θ_i from other faults:

$$Q_i \ \in \mathcal{N}(T_{3,i:}) \backslash \mathcal{N}(T_{3,j:}),\ j \neq i \tag{24}$$

For a given i and j, the nullspace $\mathcal{N}(T_{3,i:})$ might be included in $\mathcal{N}(T_{3,j:})$, i.e.

$$\mathcal{N}(T_{3,i:}) \subseteq \mathcal{N}(T_{3,j:}).$$

for some cases. In such cases, there does not exist a non-zero Q_i that will satisfy both conditions in (17) and therefore it is not possible to isolate θ_i from θ_j. The condition that the nullspace given by (24) is not empty is equivalent with requiring that the normal rank of $T_{3,i:}$ and $T_{3,j:}$ is two, i.e.

$$normal\ rank \begin{pmatrix} T_{3,i:} \\ T_{3,j:} \end{pmatrix} = 2,\ i \neq j \tag{25}$$

(25) can be reduced further by observing that T_3 includes the full rank matrix M. Removing M from T_3 gives the following simple condition for isolation fault θ_i from θ_j:

$$normal\ rank \begin{pmatrix} G_{zu,i:} \\ G_{zu,j:} \end{pmatrix} = 2,\ i \neq j \tag{26}$$

This condition is only based on one of the matrix transfer functions G_{zu} in (1), the matrix transfer function from the control input u to the external output z, where z is the input to the parametric faults θ.

(26) gives the condition for isolation for isolation of single faults in θ. The condition can be generalized. Let the *normal rank* of G_{zu} be $l \leq min(k,m)$. The maximal number of faults that can be isolated simultaneously will be $l-1$. For isolation of faults that can occur simultaneously, the concept of group-wise faults isolation described in [13] can be applied. Here the faults are divided into groups where only fault from a single group can occur simultaneously.

Note that there is no condition on the number of measurements in the system. In general, only a single measurement signal is needed. The condition guarantee that we can design a non-zero Q_i such that the effect from the parametric fault θ_i is unchanged in the closed-loop system with the consequence that the residual vector is unchanged. However, the dimension of the control inputs u is important in this approach. This is in contrast with other fault diagnosis approaches, where the dimension of the measurement output y is important.

The isolation results given here for parametric faults are based on the passive approach where no additional inputs are applied for the diagnosis. The diagnosis is only driven by the external input d to the system. The modification of the feedback controller by a Q will change the closed-loop matrix transfer function from the external input d to external controlled output as described in [10], [16]. The closed-loop matrix transfer function will also be changed in the fault-free case. This is important for the design of Q in connection with fault detection. Here, the design of Q needs to be a trade-off between fast detection and at the same time with only a minor change of the nominal closed-loop matrix transfer function. In the isolation step, the trade-off is then between fast isolation of the detected fault and a reduction of the closed-loop performance. Here, fast isolation will in many cases have a high priority.

A. State space representation

A state space description of $T_{3,i:}$ is given in the following for the calculation of Q_i. Let the system given by (1) have the following state space representation:

$$\Sigma : \begin{cases} \dot{x} = Ax \ + \ B_w w \ + \ B_d d \ + \ B_u u \\ z = C_z x + D_{zw} w + D_{zd} d + D_{zu} u \\ y = C_y x + D_{yw} w + D_{yd} d + D_{yu} u \end{cases} \tag{27}$$

where $x \in \mathcal{R}^n$ is the state vector. Further, let F be a stabilizing state feedback gain such that $A + B_u F$ is stable. A coprime factorization of G_{yu} is then given by, [16]:

$$\begin{pmatrix} M \\ N \end{pmatrix} = \left(\begin{array}{c|c} A + B_u F & B_u \\ \hline F & I \\ C_y + D_{yu} F & D_{yu} \end{array} \right) \tag{28}$$

T_3 is then given by:

$$\begin{aligned} T_3 &= G_{zu} M \\ &= (C_z(sI-A)^{-1}B_u + D_{zu})(F(sI-A-B_uF)^{-1}B_u + I) \\ &= (C_z + D_{zu}F)(sI-A-B_uF)^{-1}B_u + D_{zu} \end{aligned}$$

Let the $i'th$ row of C_z and D_{zu} be given by $C_{z,i:}$ and $D_{zu,i:}$, respectively. Then $T_{3,i:}$ is given by:

$$T_{3,i:} \ = \ (C_{z,i:} + D_{zu,i:}F)(sI-A-B_uF)^{-1}B_u + D_{zu,i:} \tag{29}$$

and $G_{zu,i:}$ is given by:

$$G_{zu,i:} \ = \ C_{z,i:}(sI-A)^{-1}B_u + D_{zu,i:}$$

The normal rank condition given by (26) is then given by:

$$normal\ rank \left(\begin{pmatrix} C_{z,i:} \\ C_{z,j:} \end{pmatrix} (sI-A)^{-1} B_u + \begin{pmatrix} D_{zu,i:} \\ D_{zu,j:} \end{pmatrix} \right) = 2,\ i \neq j \tag{30}$$

Verifying a normal rank condition can easily be done, e.g. by computing the rank at any frequency, which is not a zero.

V. Example

The example is a spring-mass system introduced in [11]. The system includes N masses that are connected with springs and dampers. The first and last mass is connected with a spring and a damper to a fixed ground. We will use a system with 5 masses, 6 springs and 6 dampers in this paper.

Let m_i be mass no. i, k_i be the constant for spring no. i and c_i the constant for damper no. i and h_i is the control input gain to mass no. i, respectively. Further, let x_i be the position of mass no. i, u_i be the control input to mass no. i and d_i the disturbance to mass no. i.

The differential equation for the spring-mass system as well as a state space model is derived in [11]. For a system with five masses, a state space description of the system becomes:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & -\frac{c_1+c_2}{m_1} & \frac{k_2}{m_1} & \frac{c_2}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{k_2+k_3}{m_2} & -\frac{c_2+c_3}{m_2} & \frac{k_3}{m_2} & \frac{c_3}{m_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{k_3}{m_3} & \frac{c_3}{m_3} & -\frac{k_3+k_4}{m_3} & -\frac{c_3+c_4}{m_3} & \frac{k_4}{m_3} & \frac{c_4}{m_3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{k_4}{m_4} & \frac{c_4}{m_4} & -\frac{k_4+k_5}{m_4} & -\frac{c_4+c_5}{m_4} & \frac{k_5}{m_4} & \frac{c_5}{m_4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{k_5}{m_5} & \frac{c_5}{m_5} & -\frac{k_5+k_6}{m_5} & -\frac{c_5+c_6}{m_5} \end{pmatrix}$$

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$$B_u^T = \begin{pmatrix} 0 & 0 & \frac{h_2}{m_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{h_4}{m_4} & 0 & 0 & 0 \end{pmatrix}$$

$$C_y = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and $D_{yu} = 0$ when two control inputs (forces) are applied at mass no. 2 and mass no. 4 and the position of mass no. 2 is measured. The state-space description of the system is of order 10. The system is controlled by a full-order observer based feedback controller of order 10 with feedback gain F and observer gain L.

For this system, we will consider two different scenarios.

Scenario 1: Two spring faults: In the first scenario, we will assume that faults can occur in spring constant no. 1, k_1, or in spring constant no. 3, k_3. The two faults are modeled as parametric faults given by:

$$k_1(\theta_{k_1}) = k_1(1+\theta_{k_1}), \; k_3(\theta_{k_3}) = k_3(1+\theta_{k_3})$$

It is straightforward to see that with these faults, the A matrix can be written as:

$$A(\theta) = A + B_w \theta C_z \tag{31}$$

with the state space matrices B_w and C_z in (27) given by:

$$\begin{aligned} B_w^T &= \begin{pmatrix} 0 & 0 & \frac{k_1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0-\frac{k_3}{m_2} & 0 & \frac{k_3}{m_3} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ \theta &= diag(\theta_{k1}, \theta_{k3}) \\ C_z &= \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} C_{z,1} \\ C_{z,2} \end{pmatrix} \end{aligned}$$

Further, the direct matrices are given by $D_{zw} = 0$, $D_{zu} = 0$ and $D_{yw} = 0$.

In order to isolate the two faults, we need to design Q_1 and Q_2 satisfying (21). Let $T_{3,i}$ be given by (29) and let $B_u = \begin{pmatrix} B_{u,1} & B_{u,2} \end{pmatrix}$. This in turn gives the following extremely simple expression for Q_i:

$$Q_i(s) = \gamma_i \left(C_{z,i}(sI - A - B_u F)^{-1} \begin{pmatrix} -B_{u,2} & B_{u,1} \end{pmatrix} \right)^T \tag{32}$$

Here, γ_i is a design parameter that can be used to detune the closed loop systems. It won't influence the residual or the stability for the nominal system or for the situation where fault no. i has occurred, but in general it will make the detection and isolation more sensitive for any fault j, $j \neq i$, for higher values of γ_i. It is always possible to choose a value of γ_i for which all closed loop systems are stable. If chosen too large, however, some of the faulty systems ($j \neq i$) will almost certainly become unstable.

For this example, we have chosen unity masses, $m_i = 1, i = 1 \ldots 5$, and the spring and damper constants to:

$$k = \begin{pmatrix} 0.0724 \\ 0.0828 \\ 0.0769 \\ 0.0574 \\ 0.0687 \\ 0.0482 \end{pmatrix} \quad \text{and} \quad c = \begin{pmatrix} 0.0059 \\ 0.0039 \\ 0.0040 \\ 0.0042 \\ 0.0021 \\ 0.0065 \end{pmatrix} \tag{33}$$

The feedback and observer gains have been chosen as:

$$F^T = \begin{pmatrix} -0.0995 & 0.0447 \\ -1.8460 & 0.6805 \\ -1.0755 & -0.0687 \\ -1.6775 & -0.0407 \\ 0.2893 & -0.1631 \\ -1.2362 & -1.4206 \\ -0.0604 & -1.0491 \\ -0.0407 & -1.6620 \\ -0.2497 & 0.2886 \\ 0.4374 & -1.8600 \end{pmatrix} \quad \text{and} \quad L = \begin{pmatrix} -20.0746 \\ -4.8047 \\ -11.3446 \\ -14.3498 \\ -43.3641 \\ -0.3404 \\ -31.8334 \\ 6.5270 \\ -19.9999 \\ -4.4462 \end{pmatrix}$$

The two faults have been picked to 50% and 80% of the nominal values, respectively. $Q_i(s)$ has been designed according to (32) with $\gamma_1 = -650$ and $\gamma_2 = 350$.

Based on this design, the system has been simulated with each of the two faults and with each of the Q_i's. The result is shown in Fig. 4.

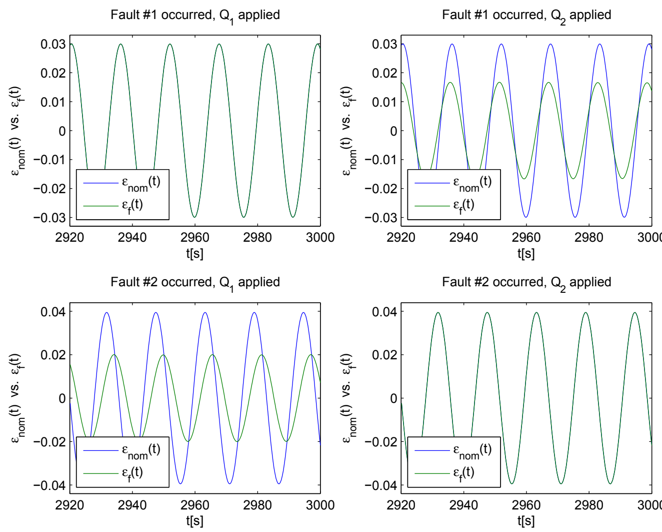

Fig. 4. Nominal and faulty residuals for faults in k_1 and k_3.

It can be seen that the residual signals in the nominal and the faulty cases are indiscernible when fault no. i has happened and when $Q_i(s)$ has been applied. However when Q_i is applied to the system when fault no. $3-i$ has occurred, the residuals change dramatically. This demonstrates that the two faults can be both detected and isolated. If the actual disturbance is not periodical, the isolation will instead have to be detected based on the spectrum of the residuals. An approach to this is to apply a bandpass filter to the frequency region where the nominal and the faulty transfer function differ the most after applying $Q_i(s)$.

Scenario 2: A spring fault and a damper fault: In the second scenario, we will assume that faults can occur in spring constant no. 3, k_3, or in damping constant no. 5, c_5. The two faults are modeled as parametric faults given by:

$$k_3(\theta_{k_3}) = k_3(1+\theta_{k_3}), \; c_5(\theta_{c_5}) = c_5(1+\theta_{c_5})$$

In this case, B_w and C_z in (31) are given by:

$$\begin{aligned} B_w^T &= \begin{pmatrix} 0 & 0 & 0 & -\frac{k_3}{m_2} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{c_5}{m_4} & 0 & \frac{c_5}{m_5} \end{pmatrix} \\ \theta &= diag(\theta_{k3}, \theta_{c5}) \\ C_z &= \begin{pmatrix} 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} C_{z,1} \\ C_{z,2} \end{pmatrix} \end{aligned}$$

The same system parameters as in (33) have been picked for this scenario and the same nominal controller has been applied. The fault in k_3 has again been assumed to be at 80% of its nominal value. For the damper, however, it is assumed that the damper is almost stuck, such that its damping value has increased by a factor of 100.

The $Q_i(s)$ have again been designed by using (32). The two values of γ_i have been chosen to 810 and -170, respectively.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/LCSYS.2020.3003238, IEEE Control Systems Letters

The resulting simulation for a periodic disturbance is shown in Fig. 5.

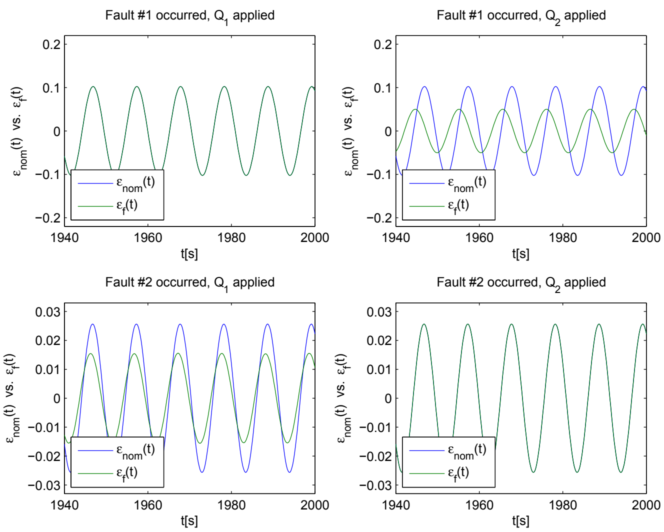

Fig. 5. Nominal and faulty residuals for faults in k_3 and d_5.

From Fig. 5, it can again be seen that the residual signals in the nominal and the faulty cases are indiscernible when Fault no. i has happened and when $Q_i(s)$ has been applied. However when Q_i is applied to the system when Fault no. $3-i$ has occurred, the residuals change dramatically. This demonstrates that the two faults can be both detected and isolated. This is actually a somewhat non-intuitive result, for as it has been reported in [11], the effect of a damping error is almost invisible in a nominal simulation. Even with the detuning by Q_1, it is hardly visible in the system output. However, as shown in Fig. 5 it becomes very distinctive in the residual signal.

Theoretically, it should be possible to isolate all three faults considered in the two scenarios above, i.e. the faults in k_1, k_3, and c_5. This is also true. However, an actual design shows that the joint sensitivity becomes so poor that while it might be possible to distinguish the three faults in simulation, it will not be possible to separate them in practice by passive fault isolation for an actual system that has modeling errors, measurement noise, etc. It seems to be possible to isolate all possible spring constant faults, but if damping errors occur, at some point, it becomes impossible to isolate all faults in practice by passive fault isolation. This is in line with the analysis results given in [11], where the fault signature matrix has been analyzed with respect to fault detection.

The simulation in this example is done without stochastic input disturbances or measurement noise. Including input disturbances and measurement noise, will not affect the decoupling result given in Sec. IV. This is a direct consequence of (13), where the input d includes both input disturbances as well as measurement noise. When input disturbances and measurement noise are included, stochastic test methods are needed for the detection of changes in the residual signals. Here CUSUM or GLR tests might be applied, [1], [2], [5].

VI. Conclusion

The problem of isolation of parametric faults in closed-loop systems has been considered in this paper. A YJBK controller architecture has been applied. The fault isolation is based on the dedicated design of the YJBK matrix transfer function. It is shown that changing the feedback controller via the YJBK matrix transfer function will result in a change in the residual output when parametric faults had occurred in the system. This is used for both fault detection and fault isolation.

Using an arbitrary non-zero stable YJBK matrix transfer function will, in general, suffice for the detection of parametric faults in the system. The isolation task is done by an active decoupling approach, where a specific YJBK matrix transfer function is designed such that it has no effect on the residual vector in the case for a specific fault and else it results in a change of the residual vector. Condition for isolability of single faults is given. The condition is simple rank condition of certain matrix transfer functions.

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